Lamb wave Propagation in Functionally Graded Piezoelectric Material Created by Internal Temperature Gradient

Yosra Dammak, Jean-Hugh Thomas, Mohamed Hèdi Ben Ghozlen

To cite this version:

Yosra Dammak, Jean-Hugh Thomas, Mohamed Hèdi Ben Ghozlen. Lamb wave Propagation in Functionally Graded Piezoelectric Material Created by Internal Temperature Gradient. Physics Procedia, Elsevier, 2015, 70, pp.122-125. 10.1016/j.phpro.2015.08.057 . hal-02514161

Available online at www.sciencedirect.com

ScienceDirect

Physics Procedia 70 (2015) 122 – 125

Physics Procedia

2015 International Congress on Ultrasonics, 2015 ICU Metz

Lamb wave Propagation in Functionally Graded Piezoelectric Material Created by Internal Temperature Gradient

Y. Dammak[a,b]*, J. H. Thomas[a], M.H. Ben Ghozlen[b]

[a]*Université du Maine, LAUM, CNRS UMR 6613, Avenue Olivier Messiaen, 72085 Le Mans, France*
[b]*Université du Sfax, Laboratoire de Physique des Matériaux, Faculté des Sciences de Sfax, 3018 Sfax, Tunisia.*

Abstract

This work presents a theoretical study of the propagation behavior of lamb wave in a functionally graded piezoelectric material (FGPM). The piezoelectric material is polarized when the six fold symmetry axis is put along the propagation direction x1 and the material properties change gradually perpendicularly to the plate. The FGPM behavior is created by forming a temperature variation across the plate.The ordinary differential equation (ODE) and the Stiffness Matrix Method (SMM) are used to investigate the propagation of the lowest-order symmetric (S_0) and antisymmetric (A_0) Lamb wave modes.

*Keywords:*Lamb Wave; FGPM; Stiffness Matrix Method

---

1. Introduction

The research of surface acoustic wave propagation problems related to FGPM have been widely studied. Some works are mainly based on analytical approaches as WKB (Wentzel-Kramers-Brillouin) method which is used to solve the coupled electromechanical field differential equation of love wave propagation in FGPM materials. The wave propagation model utilizes a unique gradient coefficient meaning that the electrical and mechanical parameters vary in the same way in term of thickness [1]. Different numerical procedures have been also used as Hybrid and Stiffness matrix concerning propagation of Saw in multilayered and FGPM systems. These methods make necessary the discretization of the graded structure into sublayers of averaged properties in the direction of the compositional

---

* Corresponding author. Tel.: +33-671-671-445.
E-mail address:yosra.dammek.etu@univ-lemans.fr

doi:10.1016/j.phpro.2015.08.057

gradient [2], and the constants distribution are unrealistic, they are all distributed with a defined mathematical function (linear, exponential, Gaussian or quadratic) [3], because it is difficult to estimate material property changes synchronously in terms of a certain law [4].

2. Theoretical background

2.1. Problem Statement

In the present work, the study is based on the differential equations associated with a FGPM plate written according the Thomson-Haskell parameterization of the Stroh formalism. The Stiffness matrix method is employed for solving the problem of Lamb wave in a FGPM plate of Lead Zirconate Titanate Ceramic (PZT-4D), with thickness labeled h, equal to 1,6 mm. The inhomogeneity is created by forming a temperature gradient along the plate (the x_3 axis) and each material constants, piezoelectric, dielectric, and elastic has a different variation's profile.The (x_1,x_2,x_3) is the rectangular Cartesian coordinates and the (x_1, x_2) plane is parallel to lamb wave polarization, and the PZT six fold symmetry axis is put parallel to the propagation direction x_1, so that the Lamb mode becomes piezoactive.

2.2. Governing differential equations

The system of governing equations of the piezoelectricity, in the absence of body force and free charges, are written below [5]:

$$\rho \ddot{u}_i = \tau_{ij,j}, \qquad D_{i,i} = 0 \qquad \text{(i=1, 3)} \tag{1}$$

$$\tau_{ij} = c_{ijkl}u_{k,l} + e_{lij}\phi_{,l}, \qquad D_i = e_{ikl}u_{k,l} - \varepsilon_{il}\phi_{,l} \qquad \text{(i=1, 3)} \tag{2}$$

Where u_j are the mechanical displacement components, ϕ the electrical potential function, and ρ is the density. In the same way τ_{ij} and D_j denote the stress and electric displacement field respectively. The remaining magnitudes c_{ijkl}, e_{jk}, ε_{kl} are the elastic, piezoelectric and the electric constants, respectively. To describe the Lamb wave in FGPM, the following boundary and continuous conditions should be satisfied shorted and open circuit together with the mechanical free surface condition on both sides (x_3= 0, -h):
Shorted circuit: $\phi=0$ and $\tau_{i3}=0$ Open circuit: $\sigma=0$, $\tau_{i3}=0$
σ refers to the charge density, it can be deduced from the normal electric displacement component values in the neighborhood of the plate free surface:

$$\sigma(-h) = D_3(-h^+) - D_3(-h^-) \tag{3}$$

$$\sigma(0) = D_3(0^+) - D_3(0^-) \tag{4}$$

We consider the problem of plane wave propagating along the x_1 axis of the form $\xi(x_3)\exp[j(k_1x_1-\omega t)]$ where k_1 is the horizontal wave vector component, ω is the angular frequency. ξ is the state vectors defined as:
$\xi = [U, T]^T$, $U = (-j\omega u_i, -j\omega\phi)$ and $T = (\tau_{i3}, D_3)$.
The eight-component state vector for FGPM is utilized to write constitutive equations and motion equation under Thomson-Haskell parameterization of the Stroh formalism [1]:

$$\frac{d}{dx_3}\xi(x_3) = Q(x_3)\xi(x_3) \tag{5}$$

The Q(x_3), fundamental acoustic tensor (6×6), is a function of material properties, wave number and frequency.

2.3. Implementation for a functionally graded piezoelectric material

In this study, all the material properties vary proportionally to the temperature variation created in the thickness direction of the piezoelectric plate in the range from 0°C to 100°C (Fig.1). Through this plate, constants distributions are realistic [6] and it would allow us to comprehend the influence of temperature gradient upon the characteristics of wave propagation. The variation of the density in the FGPM plate is so slight that it can be disregarded [7].

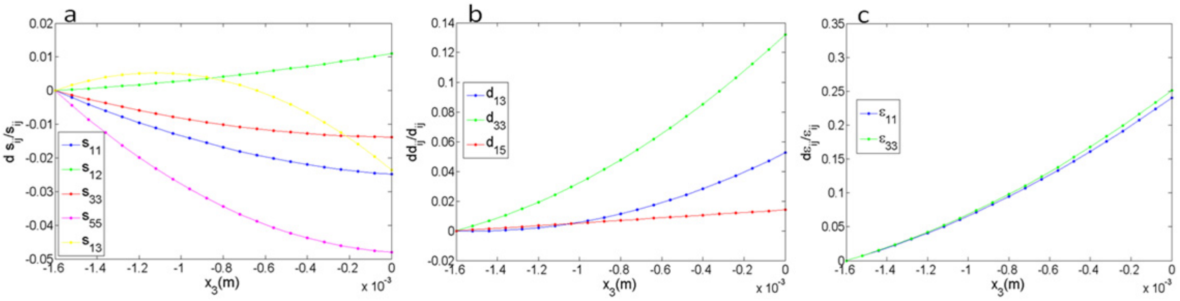

Fig. 1 Relative distribution model of the elastic (a), piezoelectric (b) and dielectric (c) constants.

3. Results

3.1. Effect of the gradient on the phase velocity

Figure 2 (a, b, c, d) show the effect of gradient on phase velocity as function of non-dimensional wave number h/λ. It can be seen that the phase velocity of S_0 Lamb mode starts at a high value, and when the frequency increases, the phase velocity decreases and becomes near V_R (the non-dispersive Rayleigh velocity). The A_0 mode performs differently, the phase velocity starts from zero value and becomes close to the same value V_R as the frequency increases. The phase velocity value of the lowest-order symmetric (S_0) and anti-symmetric (A_0) Lamb wave for the electrically open and shorted cases are also obtained denoted V_{oc} and V_{sc} respectively. The temperature gradient does not change the shape of the curves but introduces a jump in the dispersion curves compared to homogenous results; this variation is more significant in the symmetric mode (Fig. 2.a.b).

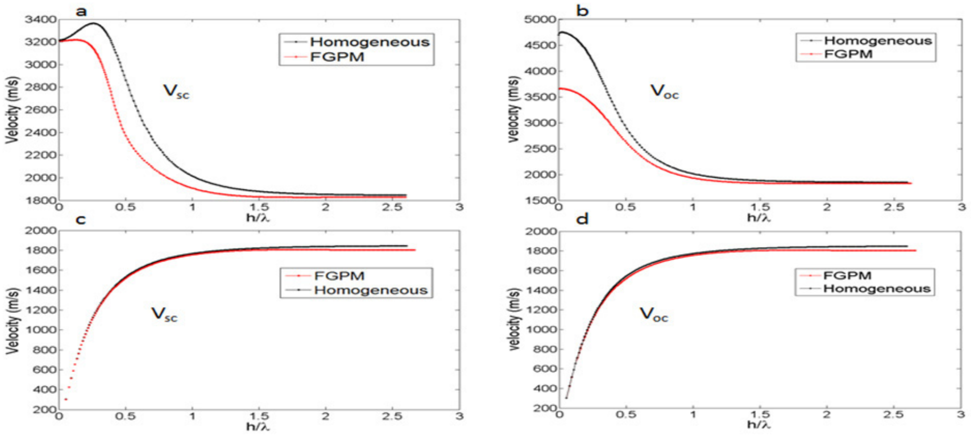

Fig. 2 Dispersion curves of S_0 mode (a) shorted circuit (b) open circuit, and A_0 mode (c) shorted circuit (d) open-circuit.

3.2. Effect of the gradient on the electromechanical coupling factor

For surface acoustic waves devices, a high electromechanical coupling factor of the wave is expected in engineering applications, it is defined as follows [9]:

$$k^2 = \frac{2(V_{oc} - V_{sc})}{V_{oc}} \tag{6}$$

The electromechanical coupling factor is presented in Fig.3. It can be found that the electromechanical coupling factor of the S_0 decreases with 64% in the FGPM case, same for the A_0 mode, with 88%.

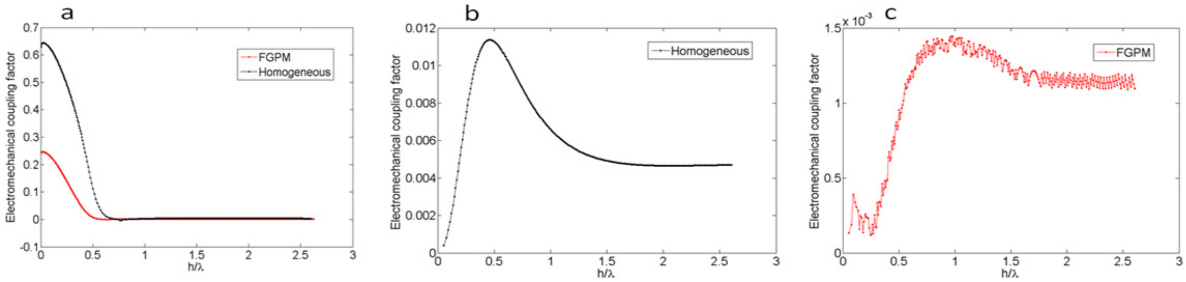

Fig. 3 Electromechanical coupling factor for S_0 mode (a) and for A_0 mode (b), (c).

4. Conclusion

The propagation of lamb wave in functionally graded piezoelectric material (FGPM) layer is investigated by a numerical matrix method. The coupled electromechanical field equations are solved exactly for the dispersion curves under both electrically open and shorted conditions. The numerical approach exhibits a high stability and a good convergence with the stratification of the plate. The effect of the temperature gradient on phase velocity and electromechanical coupling factor are plotted and discussed, the phase velocity of S_0 mode decreases enormously compared to A_0 mode in the FGPM case. In addition, the electromechanical coupling factor has a significant variation due to internal temperature gradient.

References

[1] Ben Salah, I., Wali, Y., Ben Ghozlen, M.H., Love Waves in Functionally Graded Piezoelectric Materials by Stiffness Matrix Method. Ultrasonics 51(2011) 310-316.

[2] Ming Lin, C., Yu Chen, Y., Pisano, A.P., Theoretical Investigation of Lamb Wave Characteristics in ALN/3C-SiC Composite Membranes, App. Phys. 97 (2010) 493-506.

[3] Ben Salah, I., Njeh, A., Ben Ghozlen, M.H., A Theoretical Study of the Propagation of Rayleigh Wave in a Functionally Graded Piezoelectric Material (FGPM). Ultrasonics 52 (2012) 306-314.

[4] Li, X.Y., Wang, Z.K., Huang. S.H., Love Waves in Functionally Graded Piezoelectric Materials. International Journal of Solids and Structures 41 (2004) 7309-7328.

[5] Benmeddour, F.,Grondel, S., Assaad, J., Moulin, E., Experimental Study of the A_0 and S_0 lamb Waves Interaction with Symmetrical notches. Ultrasonics 49 (2009) 202-205.

[6] Sferrit, S., Yang, H.D., Wiederick, H.D., Mukherjee, B.K., Temperature Dependence of the Dielectric, Elastic and Piezoelectric material constants of Lead Zirconate Titanate Ceramics. Proceeding of the International Conference on Smart materials (1999).

[7] Cao, X., Jin, F., Jeon, I., Characterization of the Variation of the Material Properties in a freestanding Inhomogeneous thin Film. Phys. Lett. A 375 (2010) 220-224.

[8] Wingqvist, G., Arapan, L., Yantchev, V., Katardjiev, I., A Micromachined Thermally Compensated Thin Film Lamb Wave Resonator for Frequency Control and Sensing Applications. J. Micromech. Microeng 19 (2009).